October 1st 2009, SoCal NEGT Symposium

Welfare Effects of Spectrum Management Regimes

Ergin Bayrak

Scholar in Residence, Annenberg School for Communication
Ph.D. Candidate, Department of Economics
University of Southern California

BACKGROUND

What is electromagnetic spectrum?

BACKGROUND

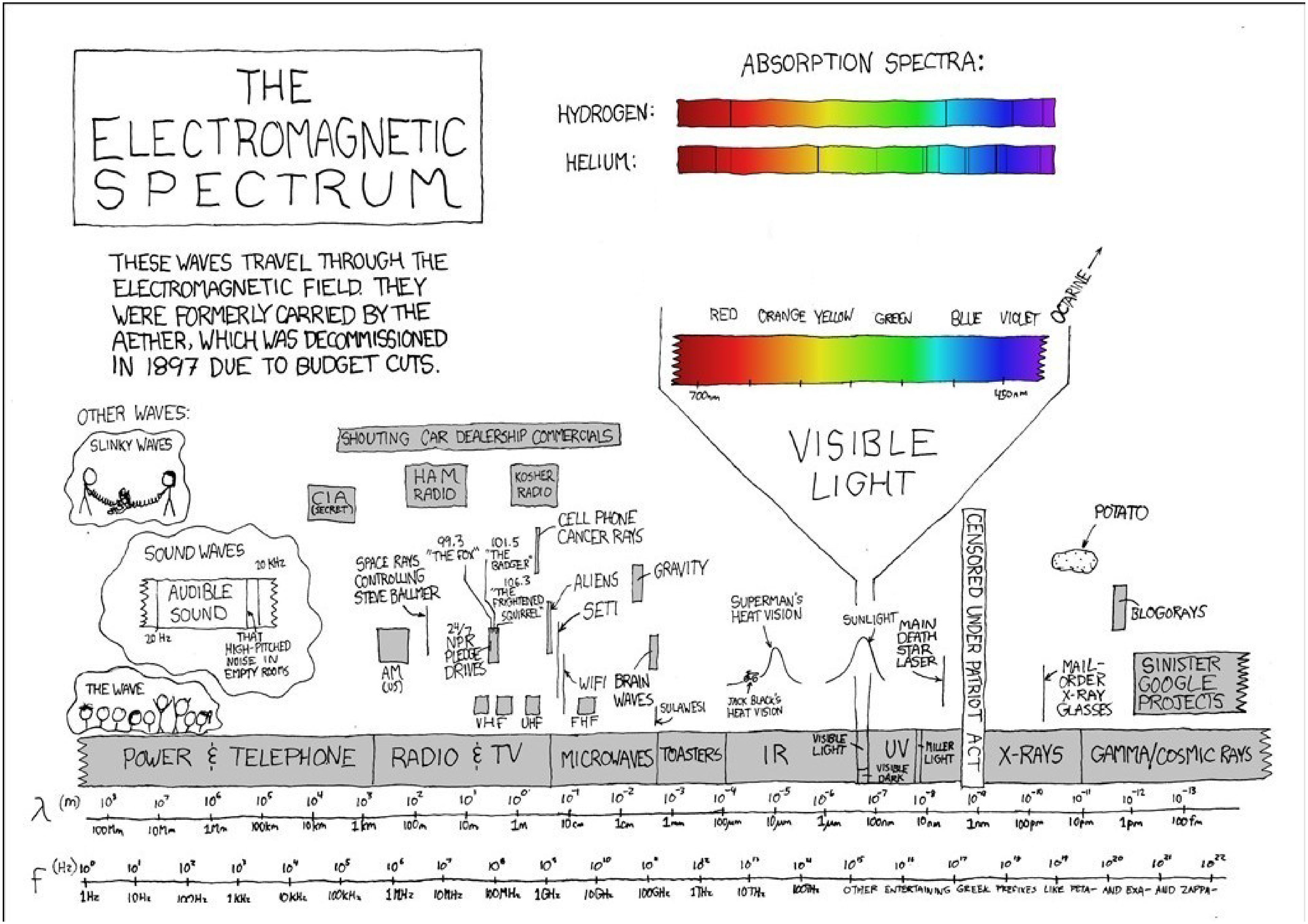
What is electromagnetic spectrum?

..Colors of light

Those we can see and those we can not

..can be utilized to carry information in the absence of physical wired connections by the use of modulation

..can be monetized


THE ELECTROMAGNETIC SPECTRUM
ABSORPTION SPECTRA:
HYDROGEN:
HELIUM:
THESE WAVES TRAVEL THROUGH THE ELECTROMAGNETIC FIELD. THEY WERE FORMERLY CARRIED BY THE AETHER, WHICH WAS DECOMMISSIONED IN 1897 DUE TO BUDGET CUTS.
RED
ORANGE
YELLOW
GREEN
BLUE
VIOLET
OCTARINE
700nm
450nm
VISIBLE LIGHT
OTHER WAVES:
SLINKY WAVES
SOUND WAVES
AUDIBLE SOUND
20 Hz
20 KHz
THAT HIGH-PITCHED NOISE IN EMPTY ROOMS
THE WAVE
SHOUTING CAR DEALERSHIP COMMERCIALS
CIA (SECRET)
HAM RADIO
KOSHER RADIO
CELL PHONE CANCER RAYS
99.3 "THE FOX"
101.5 "THE BADGER"
106.3 "THE FRIGHTENED SQUIRREL"
SPACE RAYS CONTROLLING STEVE BALLMER
24/7 NPR PLEDGE DRIVES
AM (US)
ALIENS
SETI
GRAVITY
WIFI
BRAIN WAVES
SULAWESI
VHF
UHF
FHF
SUPERMAN'S HEAT VISION
JACK BLACK'S HEAT VISION
SUNLIGHT
MAIN DEATH STAR LASER
CENSORED UNDER PATRIOT ACT
POTATO
BLOGORAYS
MAIL-ORDER X-RAY GLASSES
SINISTER GOOGLE PROJECTS
POWER & TELEPHONE
RADIO & TV
MICROWAVES
TOASTERS
IR
VISIBLE LIGHT
UV
VISIBLE DARK
MILLER LIGHT
X-RAYS
GAMMA/COSMIC RAYS
λ (m)
100Mm
10Mm
1Mm
100km
10km
1km
100m
10m
1m
10cm
1cm
1mm
100μm
10μm
1μm
100nm
10nm
1nm
100pm
10pm
1pm
100fm
f (Hz)
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz
10MHz
100MHz
1GHz
10GHz
100GHz
1THz
10THz
100THz
OTHER ENTERTAINING GREEK PREFIXES LIKE PETA- AND EXA- AND ZAPPA-

BACKGROUND

What is electromagnetic spectrum?

..Colors of light

Those we can see and those we can not

..can be utilized to carry information in the absence of physical wired connections by the use of modulation

..can be monetized

BACKGROUND

What is electromagnetic spectrum?

..Colors of light

Those we can see and those we can not

Why is its management so important?

BACKGROUND

What is electromagnetic spectrum?

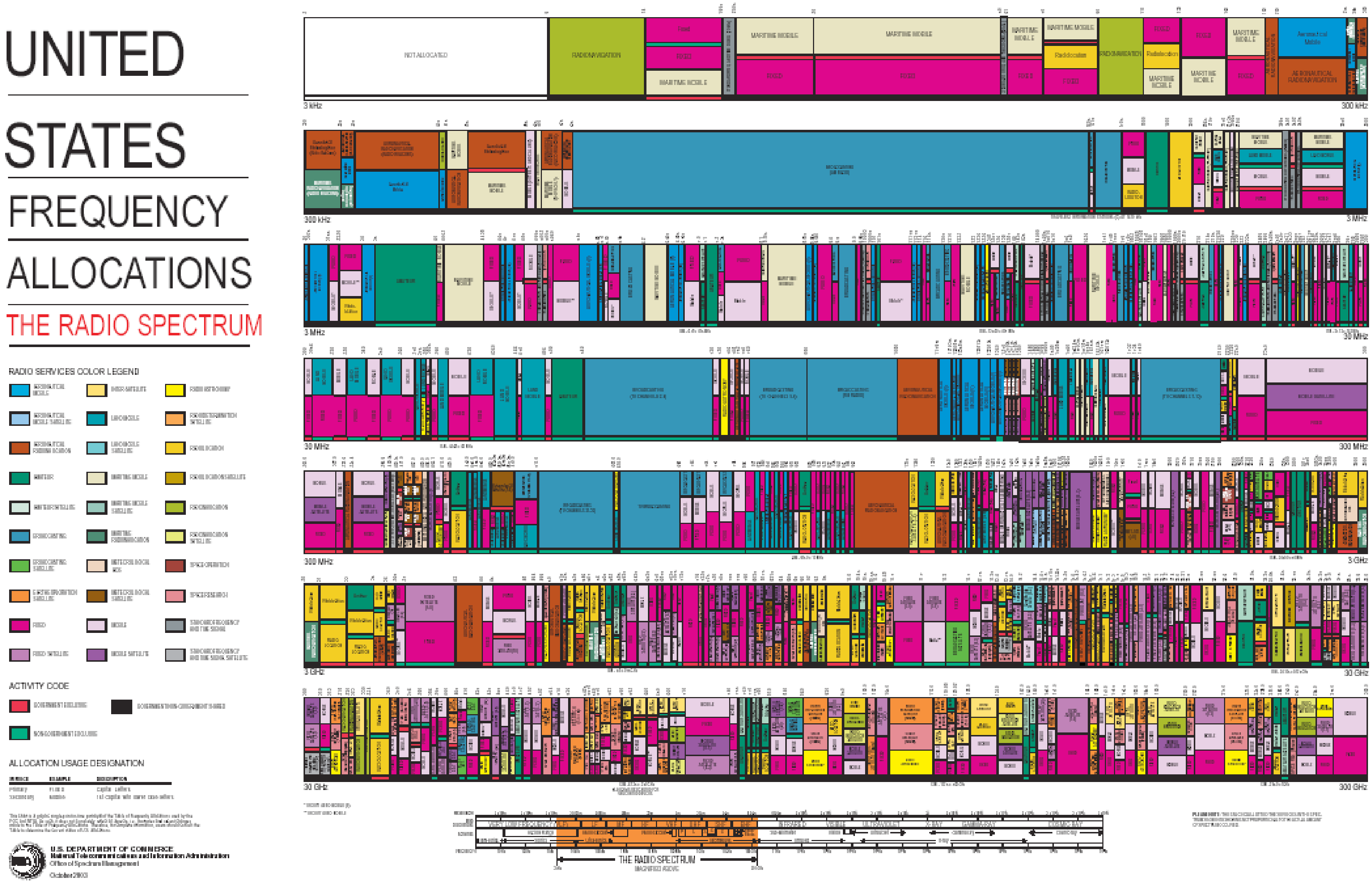
..Colors of light

Those we can see and those we can not

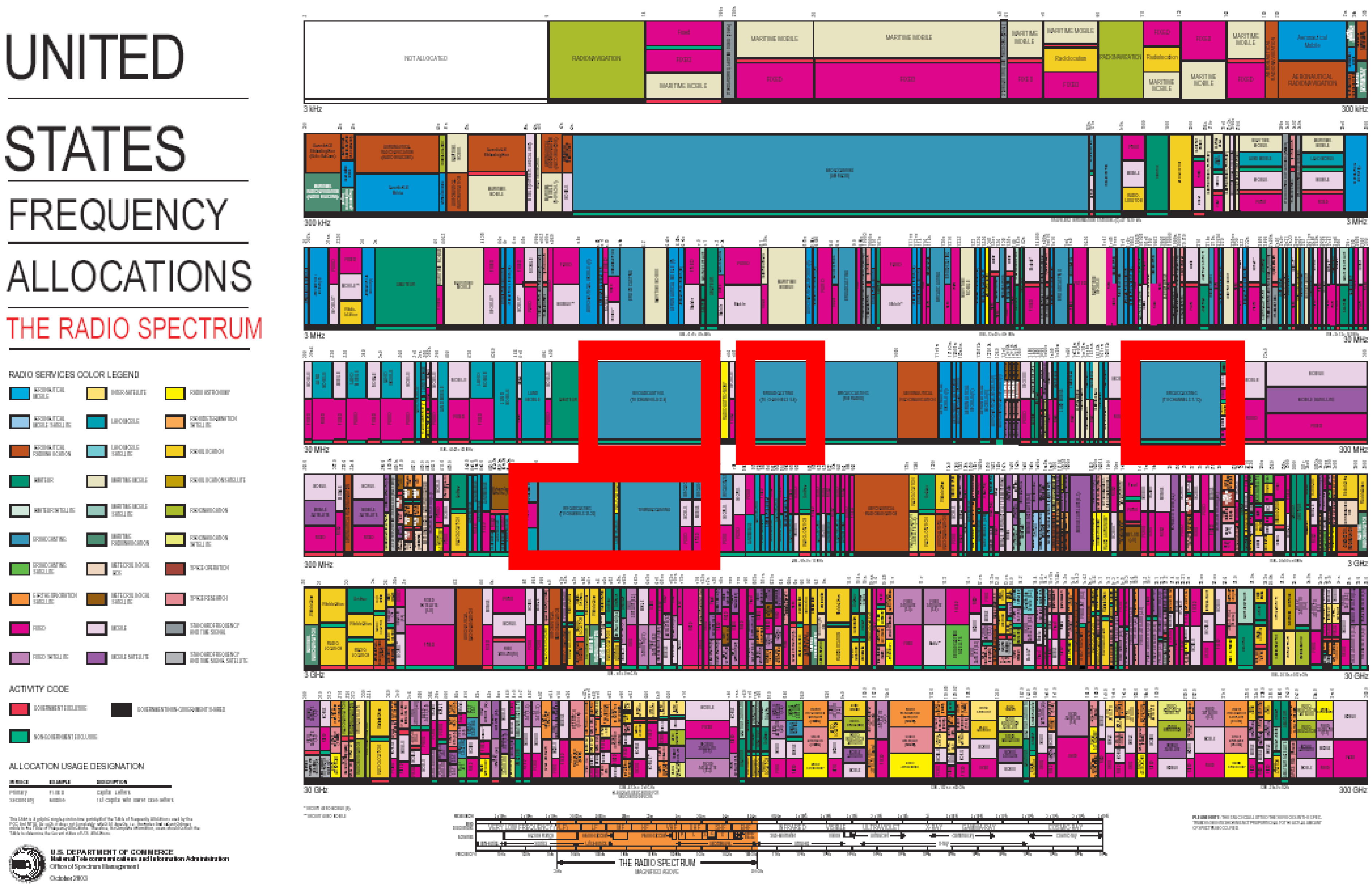
Why is its management so important?

You	20-20000Hz
Me	100-1000Hz
KCRW, KPFK, Clear Channel	88-107MHz
FOX, CNN, NBC	54-698Mhz
Cell Phone	850-1800-1900Mhz
Garage door opener	300-400Mhz
Wi-Fi/Bluetooth/Microwave	2.4-2.5GHz
Baby monitor	49Mhz
Police radar	30GHz

BACKGROUND


UNITED
STATES
FREQUENCY
ALLOCATIONS
THE RADIO SPECTRUM
RADIO SERVICES COLOR LEGEND
ACTIVITY CODE
ALLOCATION USAGE DESIGNATION
U.S. DEPARTMENT OF COMMERCE
National Telecommunications and Information Administration
Office of Spectrum Management
October 2003
NOT ALLOCATED
RADIONAVIGATION
MARITIME MOBILE
FIXED
3 kHz
300 kHz
3 MHz
30 MHz
300 MHz
3 GHz
30 GHz
300 GHz
THE RADIO SPECTRUM

BACKGROUND


UNITED
STATES
FREQUENCY
ALLOCATIONS
THE RADIO SPECTRUM
RADIO SERVICES COLOR LEGEND
ACTIVITY CODE
ALLOCATION USAGE DESIGNATION
U.S. DEPARTMENT OF COMMERCE
National Telecommunications and Information Administration
Office of Spectrum Management
October 2003
3 kHz
300 kHz
3 MHz
30 MHz
300 MHz
3 GHz
30 GHz
300 GHz
THE RADIO SPECTRUM

BACKGROUND

What to do with these ***white spaces*** ?

Licensing | **Commons**

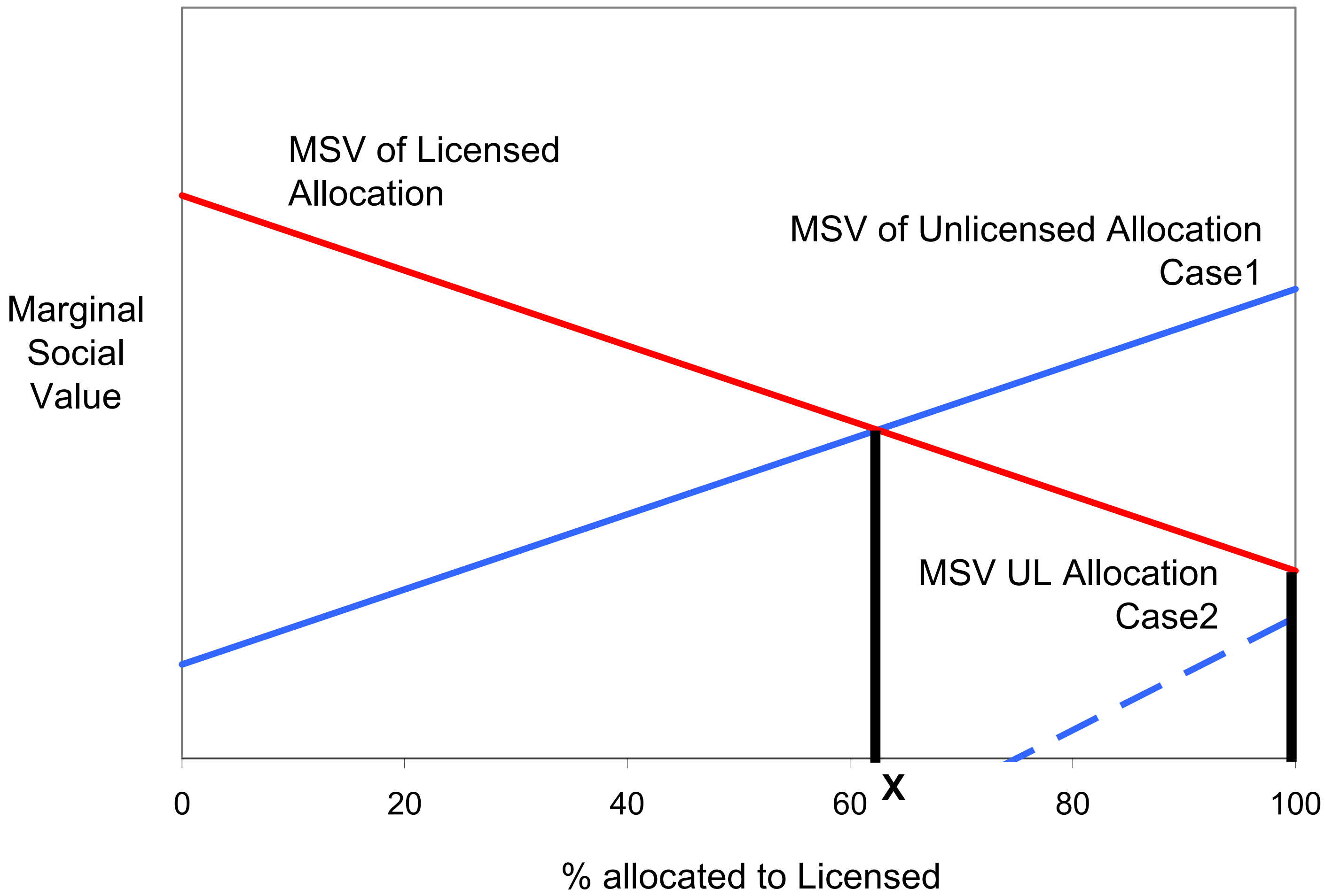
Exclusive Licenses | Unlicensed Common Access

ISSUES:

Interference
Incentives
Competition
Diversity

Consumer Welfare

Simple economics of resource allocation:



QUESTIONS

What is the social value of incremental allocations?

Is it commensurate under alternative management regimes?

Is it sensitive to non-market considerations, particularly interference?

CHALLENGES

Estimating welfare derived from unlicensed spectrum is challenging

- Used by numerous devices and services (NPV of use)
- Not traded in the usual sense (expenditure)

Estimating welfare derived from time intensive goods is challenging

- Market expenditure is miniscule compared to time use
- Time use and opportunity cost of time hard to observe

Incorporating interference and endogenous quality is challenging

- Aligning physics and economics of communication devices
- Spanning the ever increasing parameter space

CONTRIBUTIONS

A first back of the envelope estimate of welfare from unlicensed spectrum

A first model of communications market incorporating interference

PART ONE

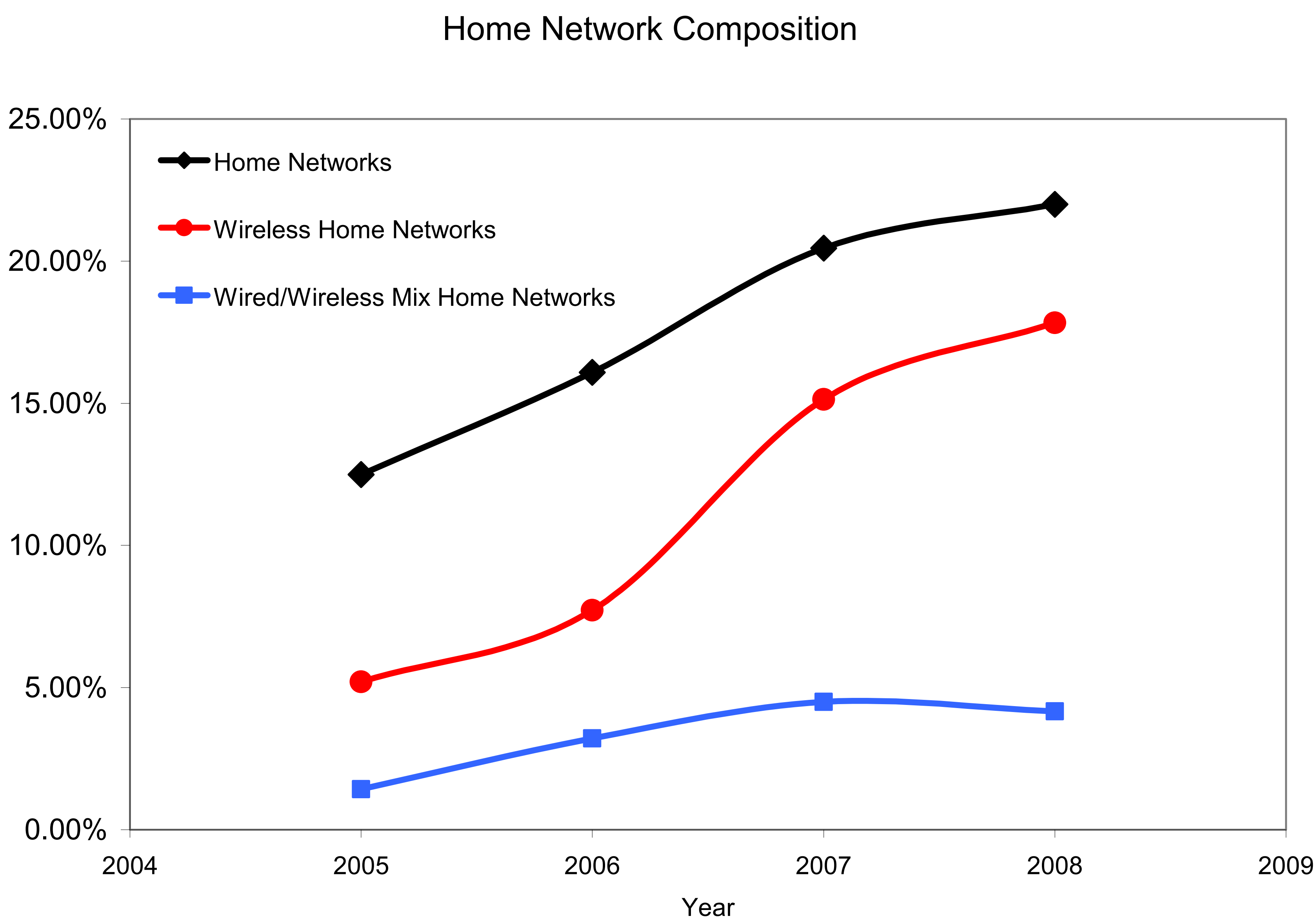
Estimate the welfare derived from *the Internet* by **wired network** owners

Estimate the welfare derived from *the Internet* by **wireless network** owners

Difference can be attributed to unlicensed spectrum (lower bound)

The time intensive nature of internet consumption:

	Market Exp.	Time
Wireless network owners	0.33% of Income	9.4%
Wired network owners	0.33% of Income	9.7%


Home Network Composition
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
2004
2005
2006
2007
2008
2009
Year
Home Networks
Wireless Home Networks
Wired/Wireless Mix Home Networks

MODEL

Consumers

$$\max U = \theta(C_i^{\alpha} L_i^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\theta)(C_o^{\beta} L_o^{1-\beta})^{\frac{\sigma-1}{\sigma}}$$

s.t.

$$P_i\, C_i + F + P_o\, C_o = W\,(\,1 - L_i - L_o\,)$$

Let

$$Y_i = (C_i^{\alpha} L_i^{1-\alpha}) \qquad\qquad Y_o = (C_o^{\beta} L_o^{1-\beta})$$

$$\rho_i = \left(\frac{P_i}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha} \qquad\qquad \rho_o = \left(\frac{P_o}{\beta}\right)^{\beta}\left(\frac{W}{1-\beta}\right)^{1-\beta}$$

Optimal Choices:

$$Y_i = \frac{W - F}{\rho_i(1+\Delta)} \qquad Y_o = \frac{W - F}{\rho_o(1+1/\Delta)}$$

where

$$\Delta = \left(\frac{\rho_i}{\rho_o}\right)^{\sigma-1}\left(\frac{1-\theta}{\theta}\right)^{\sigma}$$

Breaking down the bundles

$$C_i = \frac{\alpha\,\rho_i\,Y_i}{P_i} \qquad C_o = \frac{\beta\,\rho_o\,Y_o}{P_o}$$

$$L_i = \frac{(1-\alpha)\rho_i\,Y_i}{W} \qquad L_o = \frac{(1-\beta)\rho_o\,Y_o}{W}$$

From

$$L_i = \frac{(1-\alpha)\rho_i Y_i}{W} \qquad and \qquad Y_i = \frac{W-F}{\rho_i(1+\Delta)}$$

we have

$$\Delta = \frac{(1-\alpha)(1-F/W)-L_i}{L_i}$$

using the bundle prices and rearranging

$$\Delta = \left(\frac{(P_i/\alpha)^{\alpha}(1-\beta)^{1-\beta}}{(P_o/\beta)^{\beta}(1-\alpha)^{1-\alpha}} \right)^{\sigma-1} W^{(\beta-\alpha)(\sigma-1)} \left(\frac{\theta-1}{\theta} \right)^{\sigma}$$

ESTIMATION

$$\frac{(1-\alpha)(1-F/W)-L_i}{L_i} = A\,W^{(\beta-\alpha)(\sigma-1)}\left(\frac{\theta-1}{\theta}\right)^{\sigma}$$

Assuming small flat fixed fee for internet and taking logs

$$\ln\left(\frac{1-L_i}{L_i}\right) = \ln(A) + (\beta-\alpha)(\sigma-1)\ln(W) + \sigma\ln\left(\frac{\theta-1}{\theta}\right)$$

ESTIMATION

Time intensities

$$(1-\alpha)=1-\frac{E_i}{E_i+\frac{L_i}{(1-L_i-L_o)}} \qquad (1-\beta)=1-\frac{E_o}{E_o+\frac{L_o}{(1-L_i-L_o)}}$$

	Average Internet Use	$(1-\alpha)$	$(1-\beta)$	Average full income*
Wireless Network Owners	10.66 hrs(9.5 %)	0.9877	0.6060	\$239295
Wired Network Owners	11.04 hrs(9.8 %)	0.9881	0.6045	\$190280
Wireless Network Owners (mp)	12.54 hrs(11.1 %)	0.9895	0.5986	\$234904
Wired Network Owners (mp)	12.92 hrs(11.5 %)	0.9898	0.5970	\$186762

(mp) : taking midpoints for time use calculations *: work and leisure time valued at wage

ESTIMATION

	Coefficient	Standard Error	R^2	Implied Elasticity σ
Wireless Network Owners	0.2436	0.0327	0.0182	1.6381
Wired Network Owners	0.2003	0.0404	0.0129	1.5222
Internet for Work	-0.1507	0.0334	0.0055	N/A
Wireless Network Owners (c)	0.3131	0.0452	0.1219	1.8190
Wired Network Owners (c)	0.2558	0.0568	0.1439	1.6685
Wireless Network Owners (mp)	0.1893	0.0246	0.0194	1.4841
Wired Network Owners (mp)	0.1626	0.0305	0.0149	1.4139
Internet for Work (mp)	-0.1152	0.0253	0.0056	N/A
Wireless Network Owners (c) (mp)	0.2408	0.0340	0.1275	1.6150
Wired Network Owners (c) (mp)	0.1985	0.0428	0.1511	1.5066

(c) : controlling for value of assets, education and time spent on the internet for work related reasons (mp): midpoints

Consumer Surplus measured as Equivalent Variation

$$\frac{EV}{W} = \left[\left(1 + \frac{1}{\Delta}\right)^{\frac{1}{\sigma-1}} \left(1 - \frac{F}{W}\right) \right] - 1$$

Revoking the small flat fee assumption

$$\frac{EV}{W} = \left(1 - L_i\right)^{\frac{-1}{\sigma-1}} - 1$$

With linearized demand

$$CS = \frac{L_i}{2\sigma(1 - L_i(1 - F/W))}$$

WELFARE

	σ	EV/W	EV/W (*l*)	EV/W at median income	EV/W at average income	Difference
Wireless Network Owners	1.6381	16%	3.2%	$6755	$7684	
Wired Network Owners	1.5222	22%	3.5%	$6009	$6840	$844
Wireless Network Owners(c)	1.8190	13%	2.9%	$6342	$7285	
Wired Network Owners(c)	1.6685	16%	3.2%	$5723	$6461	$824
Wireless Network Owners (mp)	1.4841	27%	4.2%	$8762	$9980	
Wired Network Owners (mp)	1.4139	34%	4.6%	$7570	$8618	$1362
Wireless Network Owners(c) (mp)	1.6150	21%	3.9%	$8404	$9642	
Wired Network Owners(c) (mp)	1.5066	26%	4.2%	$7415	$8399	$1242

(*l*): linearized (c):controlling for value of assets, education and time spent on the internet for work related reasons

Unlicensed spectrum does create considerable welfare on the order of $18billion (824*20% of Households)

PART TWO

Given that the unlicensed allocations do result in considerable welfare, lets address the interference concern.

Do unlicensed allocations lead to a tragedy of commons because of excessive interference?

MODEL

There are M consumers with the utility function defined over the n varieties of devices as

$$U = \sum_{i=1}^{n}(q_i - \frac{q_i^2}{T_i^2}) - \gamma \sum_i \sum_{j<i} \frac{q_i}{T_i}\frac{q_j}{T_j} + q_0$$

q_i	Quantity
T_i	Quality
$0 < \gamma < 2$	Substitutability
q_0	Homogenous numeraire

Following standard utility maximization leads to inverse demand:

$$p_i = 1 - \frac{2q_i}{T_i^2} - \frac{\gamma}{T_i}\sum_{j \neq i} \frac{q_j}{T_j}$$

MODEL

Quality:

$$T_i = (1 - e^{-d_i})\, C$$

d_i	Design / robustness of devices
C	Shannon's Law (Shannon-Hartley Theorem)

Considering all possible multi-level and multi-phase encoding techniques, the Shannon–Hartley theorem states that the theoretical maximum rate of clean (or arbitrarily low bit error rate) data that can be sent with a given average signal power S through a communication channel of bandwidth W subject to additive white Gaussian noise of power N, is:

$$C = W \cdot \log_2\left(1 + \frac{S}{N}\right)$$

MODEL

Quality:

$$T_i(d_i \,|\, W, w, S, N, n) = (1 - e^{-d_i})\; W \log_2\left(1 + \frac{S}{Nm^{\varepsilon}}\right)$$

W	Bandwidth of a white space (6Mhz)
S	Base signal power
N	Base noise power
m	Number of firms per channel
ε	Interference elasticity
d_i	Design

$K(d_i)$	Cost of design $(e^{d_i} - d_i - 1)$

MODEL

Timing:

Given the number and bandwidth of white spaces and the management regime

First stage: Firms choose device design d_i

Second stage: Firms compete in device market a la Cournot

MODEL

Working backwards:

Last stage:

$$\max_{q_i} \ \pi_i = M\left(1 - \frac{2q_i}{T_i^2} - \frac{\gamma}{T_i}\sum_{j\neq i}\frac{q_j}{T_j}\right)q_i - K(d_i) - F$$

implies the equilibrium quantities and prices

$$\frac{q_i^c}{T_i} = \frac{\left(aT_i - \gamma\sum_{j=1}^{n}T_j\right)}{a\,b} \qquad p_i^c = \frac{2\left(aT_i - \gamma\sum_{j=1}^{n}T_j\right)}{T_i\,a\,b}$$

Where $a = [4 + \gamma(n-1)]$ and $b = (4 - \gamma)$

MODEL

First stage profit in terms of qualities (design)

$$\max_{d_i} \pi_i(d_i \mid d_j) = \frac{2M\left(a\,T_i - \gamma \sum_{j=1}^{n} T_j\right)^2}{(a\,b)^2} - K(d_i) - F$$

where

$$T_i = (1 - e^{-d_i})\ W \log_2\left(1 + \frac{S}{Nm^{\varepsilon}}\right)$$

Substituting quality and taking the FOC:

$$\frac{4MC^2(a-\gamma)^2}{a^2b^2}(1-e^{-d_i})e^{-d_i}+\frac{4MC^2(a-\gamma)\gamma}{a^2b^2}e^{-d_i}\sum_{j\neq i}(1-e^{-d_j})=e^{d_i}-1 \quad \forall i$$

Solving the fixed point of the BR correspondence gives optimal design:

$$d_c=\frac{1}{2}\ln\left(\frac{4MC^2[4+\gamma(n-2)]}{[4+\gamma(n-1)]^2(4-\gamma)}\right)$$

CHARACTERIZATION

$$T_c = (1 - e^{-d_c})\ W \log_2\left(1 + \frac{S}{Nm^{\varepsilon}}\right)$$

$$q_c = \frac{T_c^{\,2}}{[4 + \gamma(n-1)]} \qquad\qquad p_c = \frac{2}{[4 + \gamma(n-1)]}$$

$$\pi_c = \frac{2MT_c^{\,2}}{[4 + \gamma(n-1)]^2} - (e^{d_i} - d_i - 1) - F$$

$$CS_c(n^*) = n^* M\left(q_c - \left(\frac{q_c}{T_c}\right)^2 - \frac{\gamma(n^* - 1)}{2}\left(\frac{q_c}{T_c}\right)^2 - p_c q_c \right)$$

SIMULATION

Licensing regime: n = w

Commons regime: $n \Leftarrow$ zero profit

$$\frac{S}{N}(dB) = \{0, 10, 20, 30, 40, 50, 60, 70, 80, 90\}$$

$$\gamma = \{0.1, 0.3, 0.5, ..., 1.7, 1.9\}$$

$$\varepsilon = \{0.2, 0.4, 0.6, ..., 1.8, 2\}$$

$w = 10$

$W = 6 \times 10^6 Hz$ ($6MHz$)

$M = 1$

Simulation algorithm in pseudo-code

Algorithm : Equilibria

Input: $\{w, W, SNR, \gamma, \varepsilon, F\}$

Output: design d, quality $T(d)$, quantity q, price p, profit π, consumer surplus CS

for each SNR **in the set** $\{0, 10, 20, 30, 40, 50, 60, 70, 80, 90\}$

for each ε **in the set** $\{0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\}$

for each γ **in the set** $\{0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 1.9\}$

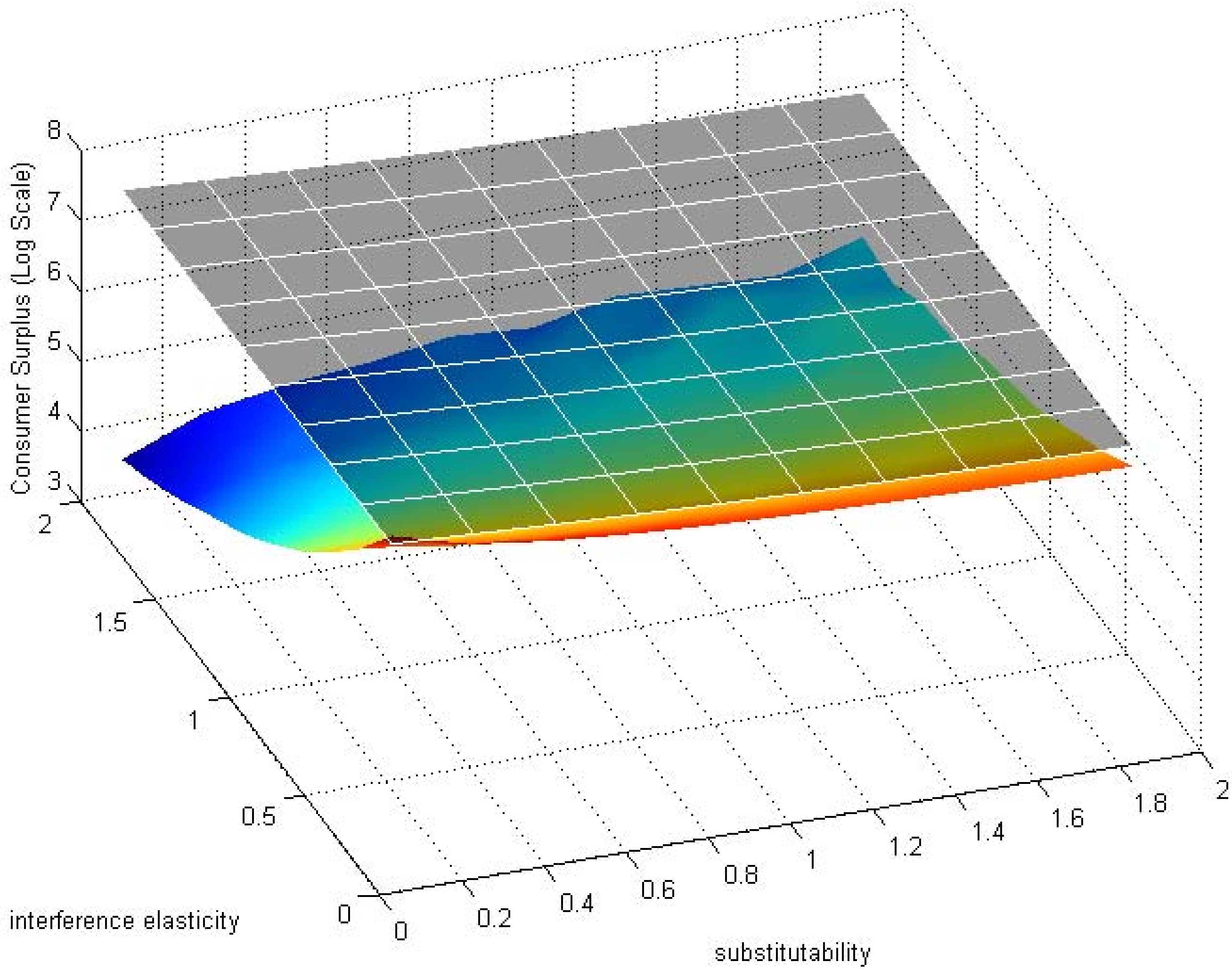
while $\pi_i > F$ for i:1,2,...nmax

calculate design d_i,quality $T_i(d_i)$, quantity q_i, price p_i, profit π_i,Consumer Surplus CS

if $\pi_i = F$ **stop**

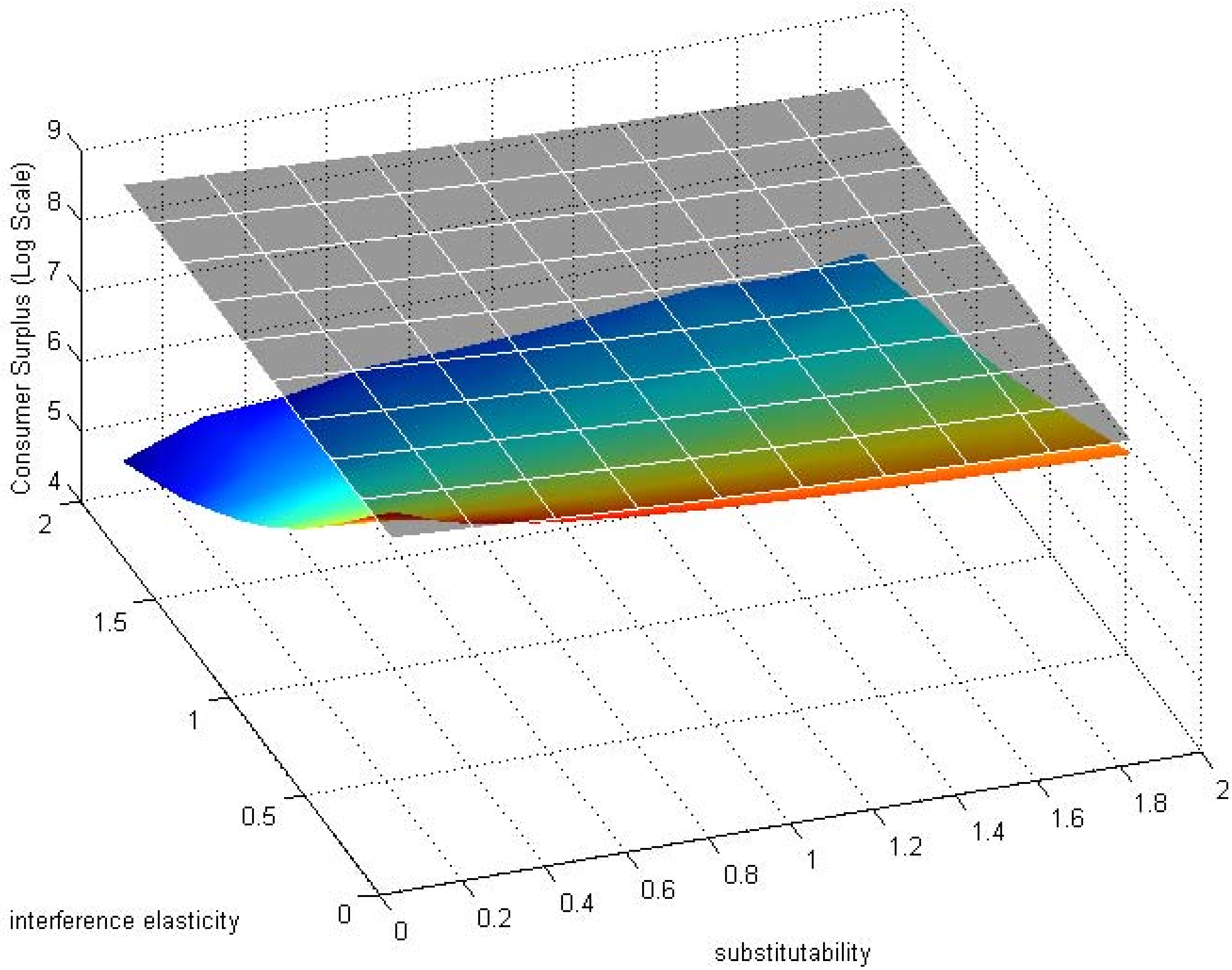
record output

terminate

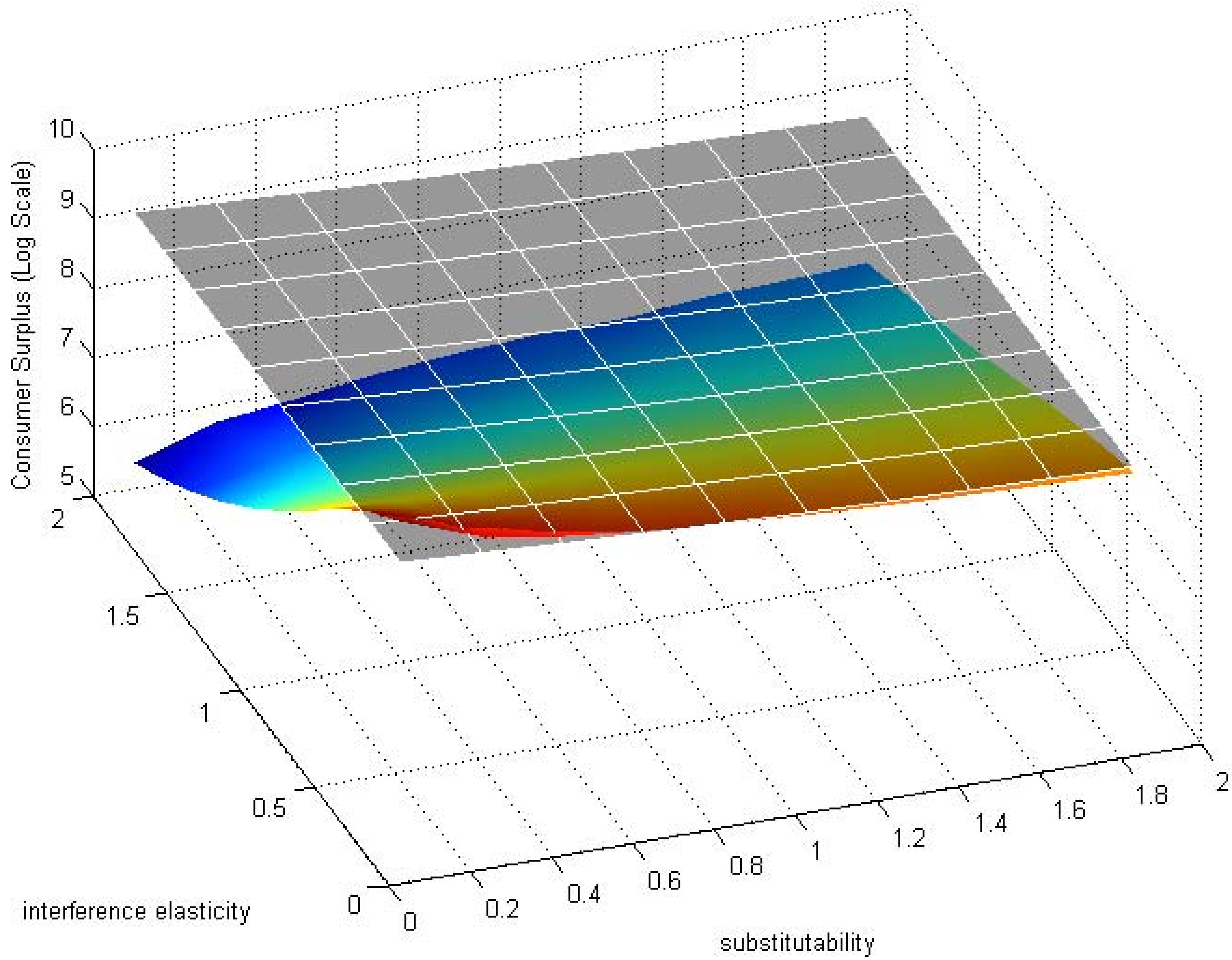


Consumer Surplus at 0dB Native SNR

RESULTS

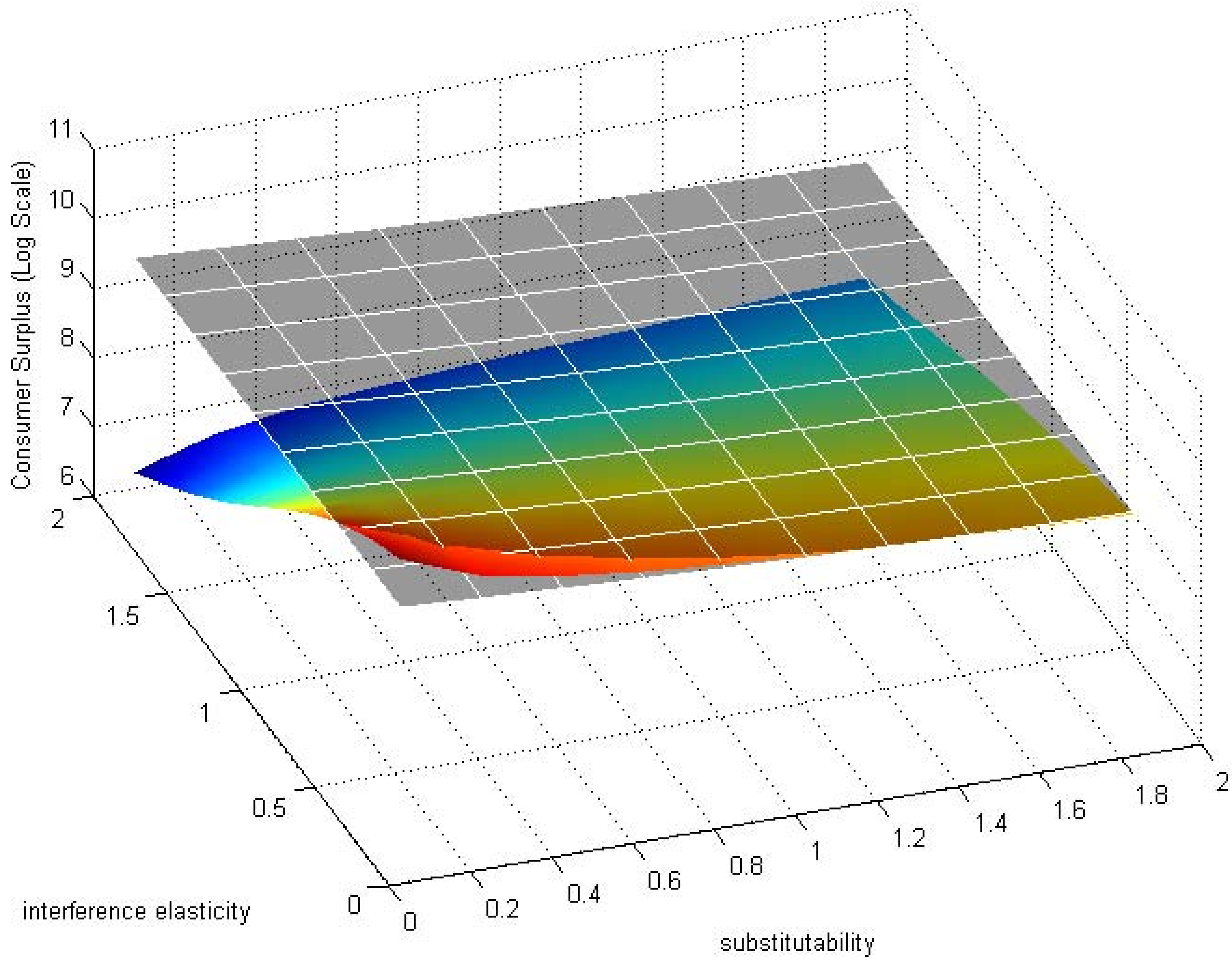


Consumer Surplus at 10dB Native SNR

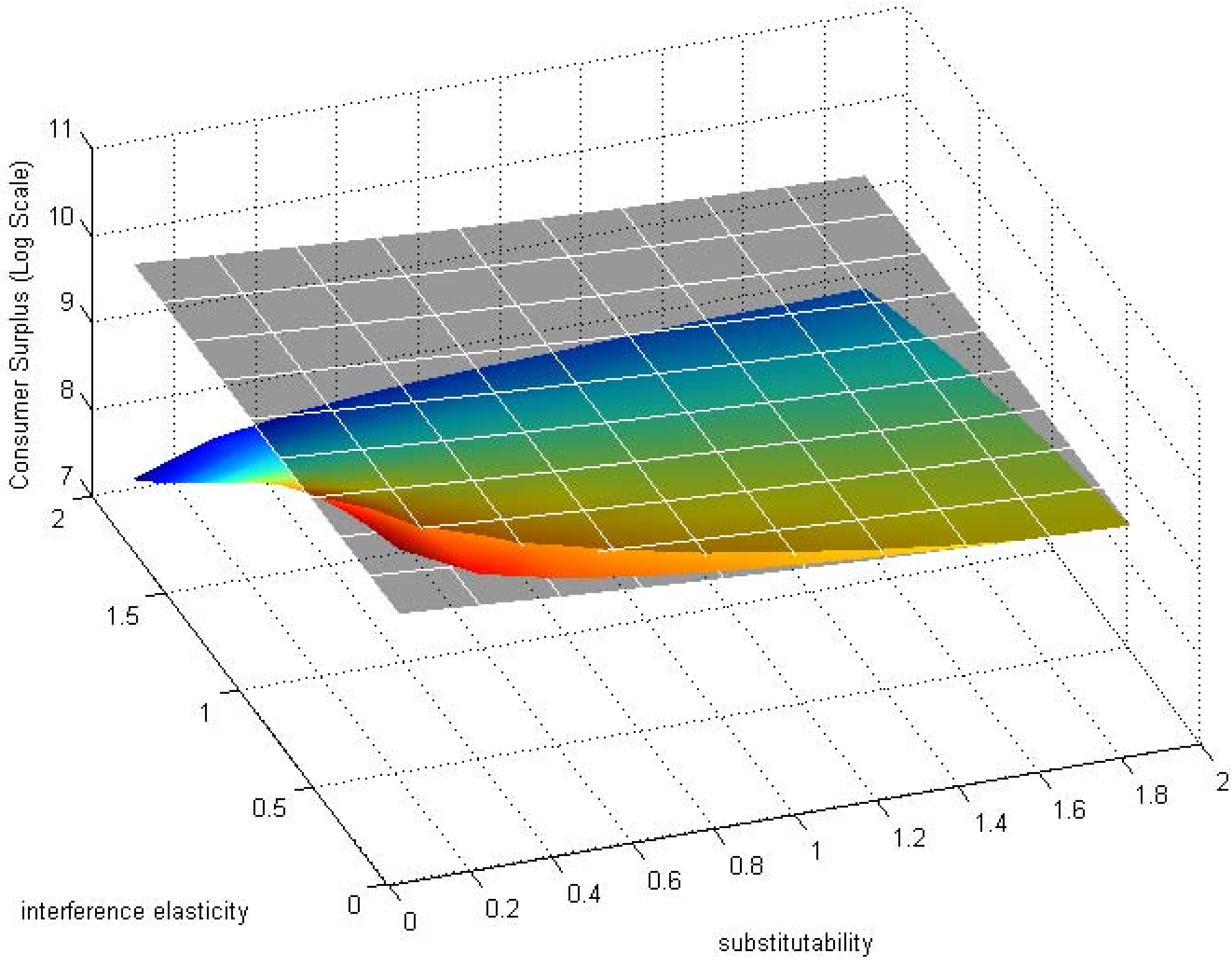


Consumer Surplus at 20dB Native SNR

RESULTS

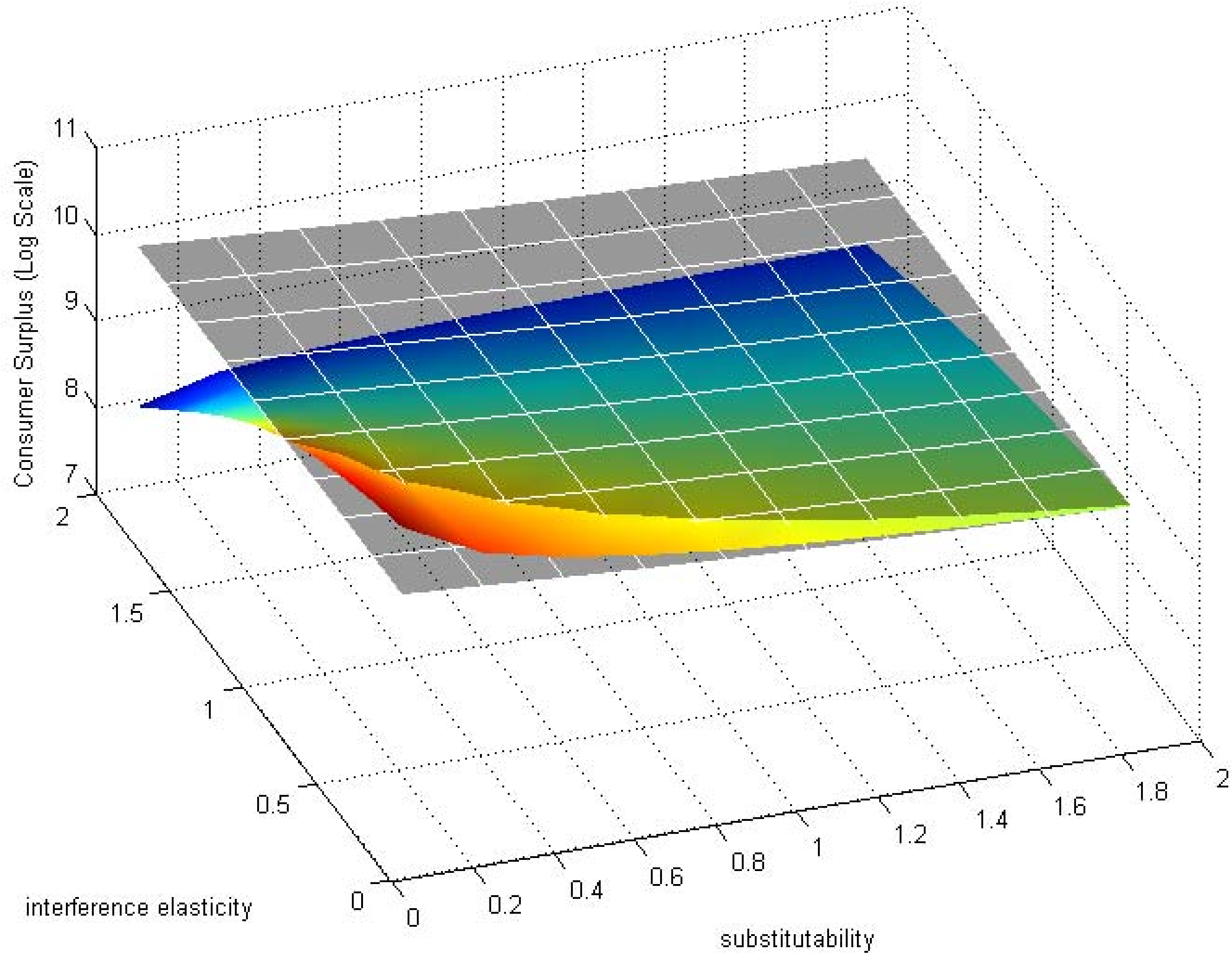


Consumer Surplus at 30dB Native SNR

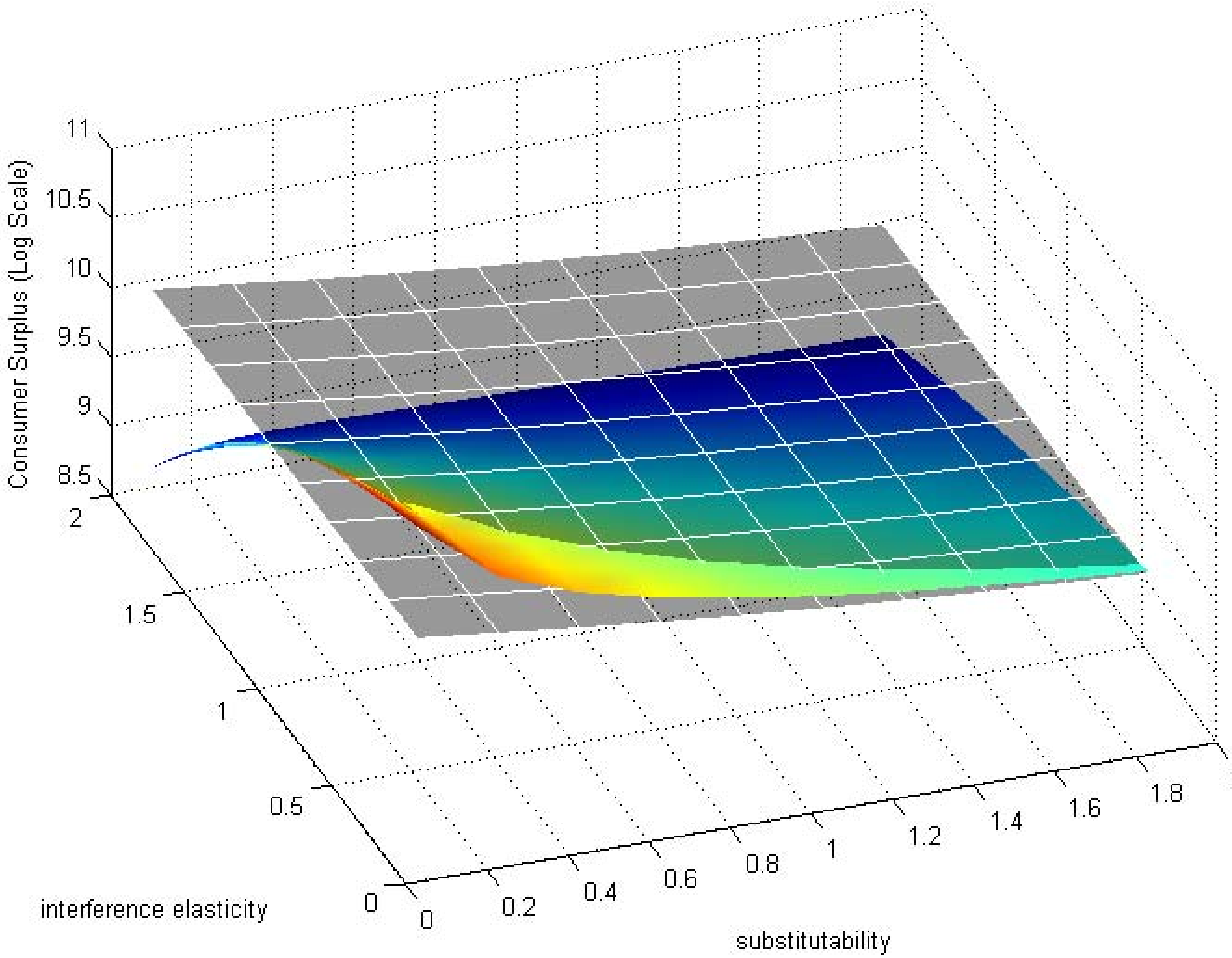


Consumer Surplus at 40dB Native SNR

RESULTS

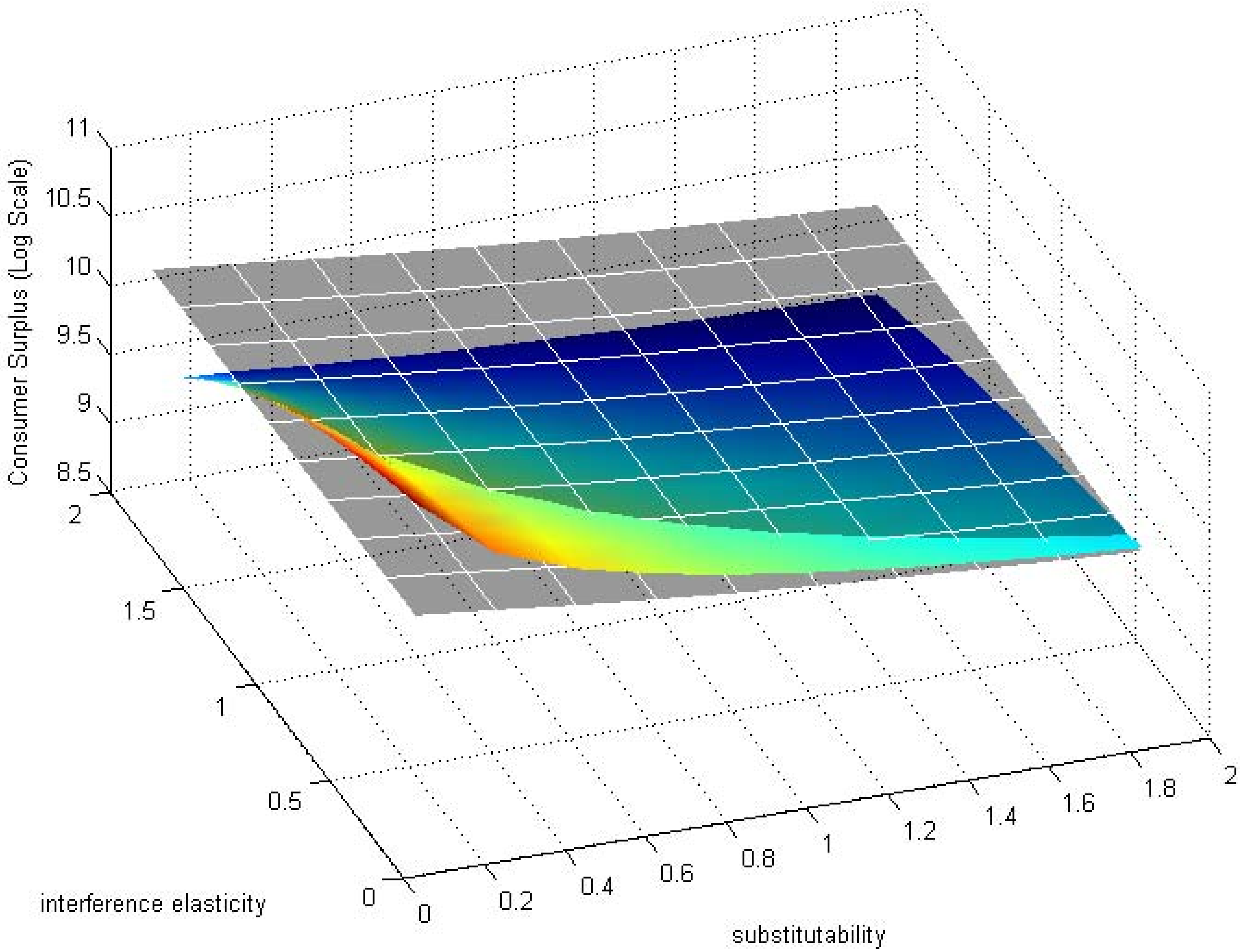


Consumer Surplus at 50dB Native SNR



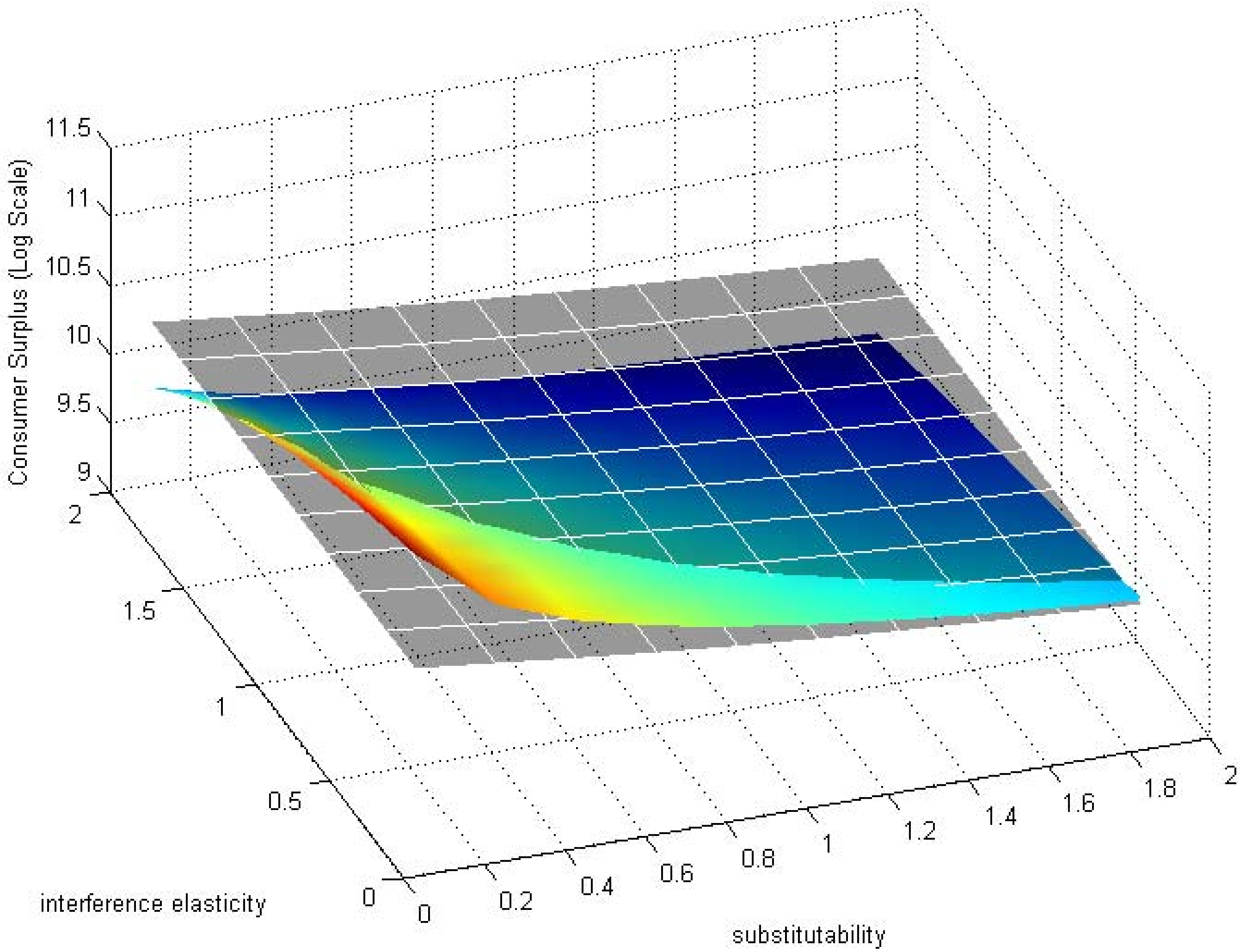
Consumer Surplus at 60dB Native SNR

RESULTS

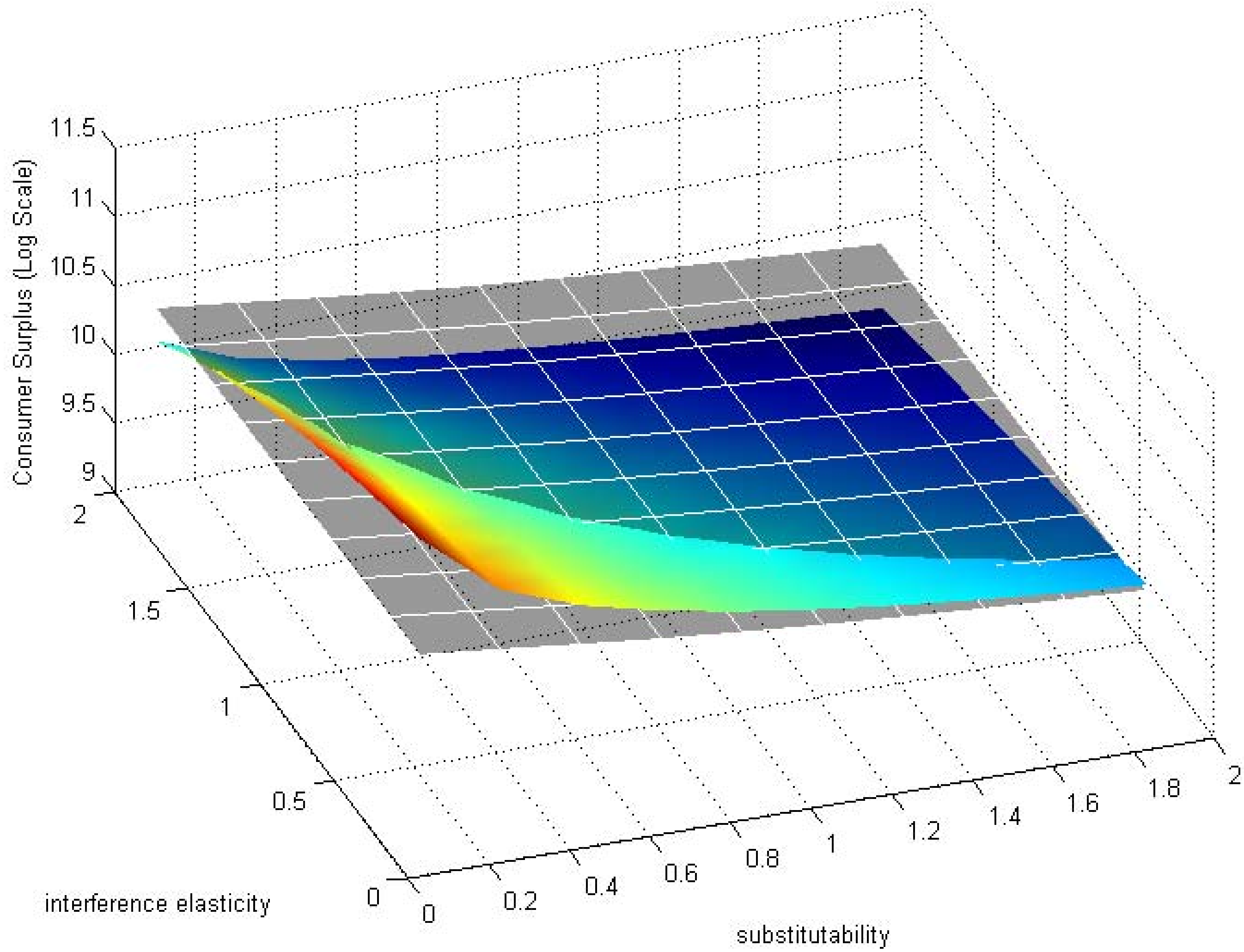


Consumer Surplus at 70dB Native SNR

RESULTS

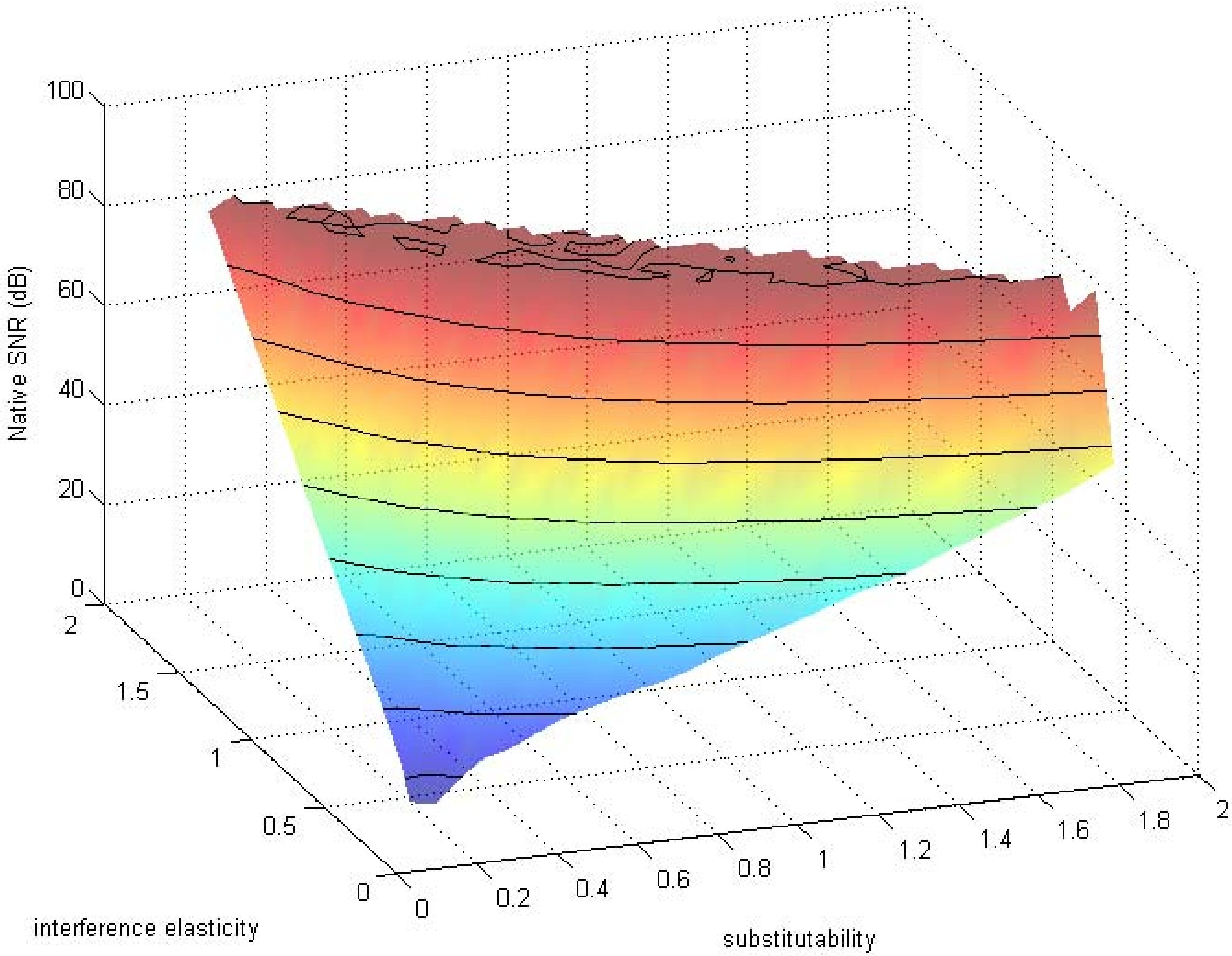


Consumer Surplus at 80dB Native SNR



Consumer Surplus at 90dB Native SNR

RESULTS



Boundary of Consumer Surplus Dominance

CONCLUSION

We have shown that unlicensed allocations do create welfare and can not be disregarded as has been done in the earlier debates on spectrum management.

We have shown that although interference degrades quality, it can lead to higher consumer surplus if the degradation is a result of differentiation. Tragedy of commons is not particularly suitable to justify licensed allocations.

All future allocations should be guided by marginal social value criterion and should be informed by consumer preferences and technological environment.

Thank You!

Ergin Bayrak

ebayrak@usc.edu